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LETTERS TO THE EDITOR

FREE VIBRATION OF CURVED PANELS WITH CUTOUTS

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1. Introduction

Some of the structural components of aircraft, missile and ship structures can be idealized as curved panels. Quite often, to save weight and also to provide a facility for inspection, cutouts are provided in these panels. Also, there can be some instruments directly fixed to these panels, and the safety of these instruments can be dependent on the vibration characteristics of the panels. Hence free vibration studies on curved panels with cutouts are of interest to structural engineers.

If the classical theory of plates is assumed to be valid, the governing differential equation for a thin plate is biharmonic and exact solutions cannot be realized for many configurations of the plate. If curvature is also present in the plate, then there is also stretching in addition to bending, and solution of such problems is obviously more difficult. Providing cutouts will enhance the difficulty of the solution. Thus, to understand the behaviour of panels with cutouts, approximate methods have to be used.

Paramasivam [1] made use of the finite difference method to study the behaviour of square plates with symmetrical square cutouts. Simply supported and clamped boundary conditions were considered for analysis. Aksu and Ali [2] studied the dynamic characteristics of rectangular plates with rectangular cutouts using variational techniques in conjunction with a finite difference method. Results were presented for plates with one as well as two cutouts; the plates were clamped on two parallel edges and simply supported on the other two. Mundkur *et al*. [3] have studied the vibration of square plates with square cutouts by using boundary characteristics orthogonal polynomials satisfying the boundary conditions along the plate outer edge and making use of the Rayleigh–Ritz method. Solutions were given for many boundary conditions.

Studies have also been carried out on curved plates. Petyt and Debnath [4] have obtained natural frequencies of rectangular singly curved plate making use of the Kantorovich method. Results were presented for plates clamped on all edges. Free vibration characteristics of a singly curved rectangular plates were obtained by making use of the extended Rayleigh–Ritz method, the finite element method and the Kantorovich method [5]. Results were presented for panels clamped on all edges. An approximate method for the determination of natural frequencies of stiffened and curved plates was suggested in reference [6]. With trigonometric functions representing the three displacements, an energy method was used to obtain the values of the frequencies for panels simply supported on all edges. This method of solution could result in an unacceptable solution for certain cases and this deficiency was discussed by Webster and Warburton [7].

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In the present analysis, the effect of cutouts on the natural frequencies of plates with some classical boundary conditions are studied. The plate has curvature in one direction and is straight in the other. This structure could be imagined as an idealization of the panel of a circular cylindrical shell supported longitudinally by stiffeners and along the circumference by bulkheads.

A 28-degree-of-freedom rectangular element is used for the study. Results obtained when specialized for flat plates agree well with those of reference [3] and the behaviour of the shell structure with cutouts seems akin to that of plates with similar cutouts, although numerically the effect of curvature is to increase the frequency multi-fold.

2. BRIEF DETAILS OF THE ELEMENT

The element chosen for study is developed by using Olson's polynomial used in the development of the Olson–Lindberg shell element [8]. The element has seven degrees of freedom per node; viz., u, $\partial u/\partial y$, v, $\partial v/\partial y$, w, $\partial w/\partial x$ and $\partial w/\partial y$.

The co-ordinate system chosen is as shown in Figure 1 and the following functions are used to represent the displacements:

$$w(x, y) = a_1 + a_2x + a_3y + a_4xy + a_5x^2 + a_6y^2 + a_7x^2y + a_8xy^2 + a_9x^3 + a_{10}y^3 + a_{11}x^3y + a_{12}xy^3, \tag{1}$$

$$u(x, y) = a_{13} + a_{14}x + a_{15}y + a_{16}xy + a_{17}y^2 + a_{18}xy^2 + a_{19}y^3 + a_{20}xy^3, \tag{2}$$

$$v(x, y) = a_{21} + a_{22}x + a_{23}y + a_{24}xy + a_{25}y^2 + a_{26}xy^2 + a_{27}y^3 + a_{28}xy^3. \tag{3}$$

The polynomial function used for the radial displacement is the same as that used for quadrilateral plate elements. Since the lines $y = 0$ and $y = b$ are straight, only a linear function for x is used for in-plane displacements; improve the account taken of the curvature effect, higher order polynomials are used for y.

The Strain–displacement relationships of classical shell theory [9] are used in the development of the stiffness matrix. They are

$$\varepsilon_{xx} = (\partial u/\partial x) - z(\partial^2 w/\partial w^2), \tag{4}$$

$$\varepsilon_{yy} = (\partial v/\partial y) + (w/R) - z\{(\partial^2 w/\partial y^2) - (1/R)(\partial v/\partial y)\}, \tag{5}$$

$$\varepsilon_{xy} = (\partial v/\partial x) + (\partial u/\partial y) - 2z\{(\partial^2 w/\partial x\,\partial y - (1/R)(\partial v/\partial x)\}, \tag{6}$$

By making use of the above expressions for the strains, the strain energy can be calculated in terms of the displacements, and by using the shape functions in terms of the nodal displacements; from this, the stiffness matrix can be obtained. It is worth mentioning that while the authors have neglected the terms which are of order h^2/R^2, in the present

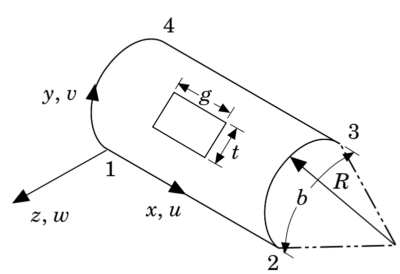

Figure 1. The shell element with the co-ordinate system.

development all terms in the strain energy expression have been considered (i.e., including the terms of order h^2/R^2). The kinetic energy also can be expressed in terms of the assumed displacements u, v and w, and from this the consistent mass matrix can be generated. Since polynomials are used for representing the displacements, closed form integrations are carried out for the development of the stiffness matrix as well as mass matrices.

3. NUMERICAL RESULTS AND DISCUSSION

By making use of the element developed, the natural frequencies of a cantilevered fan blade has been computed. This was the same numerical exercise carried out by Olson and Lindberg to check their element against values available from experiments. Computations carried out by them using a coarse mesh (4 × 4) showed some discrepancy with the experimental results. The values obtained by using the present element with the same coarse mesh is in agreement with their values for the first three frequencies and with a finer mesh (12 × 12) the values improve, which can be seen from the fact that they compare reasonably well with those obtained experimentally. However, the numerical result for the third frequency is slightly lower than that obtained experimentally. The results are presented in Table 1. It should be expected that the results should exactly match with reference values. The material chosen is reported to be steel and the numerical values of Young's modulus and density used are not given. In the present studies the following numerical values were used for computation: radius 609·6 mm; $a = b = 304{\cdot}8$ mm; $t = 3{\cdot}043$ mm, $\nu = 0{\cdot}3$; $E = 21\,000$ kg/mm^2; $\rho = 7{\cdot}8 \times 10^{-10}$ kg/mm^3. Minor variations in these values could be one of the reasons for this discrepancy. Including the terms of order h^2/R^2 was thought of as another reason for the difference. However, it was seen from the computation that the effect was only in the first decimal. The authors of reference [8] reported, that results obtained by using the element differs by 2·5% from the exact results when specialized for a plate without curvature. In the present analysis, the difference is only 1% in the fundamental frequency for a cantilever plate without curvature. A possible reason for this could be in the algorithm chosen for extraction of eigenvalues and eigenvectors and the accuracy sought in the analysis.

As the next exercise, the code developed was checked for its accuracy by solving the curved plate problem solved by Petyt and Debnath [4]. Curved plates clamped on all edges were chosen for the study. The mode shapes for the first four frequencies are the same

TABLE 1

Convergence studies on a cantilever fan blade

		Frequency (Hz)		
Mesh		Mode 1	Mode 2	Mode 3
3 × 3	Present	101·8	155·9	261·9
	Reference [8]	100·7	155·1	260·9
4 × 4	Present	94·3	148·4	250·6
	Reference [8]	94·5	147·6	255·1
10 × 10	Present	87·9	141·3	250·6
12 × 12	Present	87·6	140·8	250·3
Experiment	Reference [8]	86·6	136·5	258·9

as reported in reference [4]. The numerical values of frequencies associated with these four mode shapes also show very good agreement.

After validating the program, further numerical results were obtained for square plates with cutouts for the following boundary conditions: clamped on all edges (CCCC); simply supported on all edges (SSSS); clamped on two parallel edges and simply supported on the other two (CCSS); clamped on two parallel edges and free at the other two (CCFF); clamped on one edge and free on the other three (CFFF). (In the above abbreviations, the first two subscripts correspond to the curved edges and the last two to the straight edges.) Two values of R/t (250 and 50) were used for study. Although the use of a 12×12 mesh yielded reasonable results while validating the programme, for these further studies the panel was divided into a 20×20 mesh to model the cutouts properly.

The first six frequencies are presented in Figures 2(a)–(e). The cutout is also square and positioned symmetrically. The size of the cutout was varied and the computation was carried out for values of $(g/a) = 0{\cdot}1$–$0{\cdot}8$ (in steps of $0{\cdot}1$), where g is the edge/length of the cutout. It can be seen from the figures that the trend in the behaviour is the same as that of the plates and the solution for the plate is almost the same, as that obtained in reference [3]. By comparing the numerical values obtained for $R/t = 50$ and 250, it can be seen that the effect of the curvature is to increase the frequency. Initially, the frequency decreases marginally due to the cutout, but with the increase in the size of the cutout the effect of the curvature seems to become less, and plates with and without curvature have almost the same frequency, which is much higher than that of the plate without the cutout. This could be attributed to the fact that the mass has decreased and the stiffness, due to the edge effect, has increased considerably. This behaviour is noticed for plates supported on the four sides. For plates having two or three free edges, the effect of the cutout is to decrease the frequency.

To provide a quantitative idea about the effect of cutouts as well as curvature on the natural frequencies, two different types of non-dimensional curves are presented in Figures 3 and 4. Both figures are for square plates clamped on all edges with symmetrical square cutouts. In Figure 3, the frequency is non-dimensionalized with respect to a plate of zero curvature; in Figure 4 the non-dimensional value is with respect to the same plate (corresponding dimensions and curvature) without a cutout.

From Figure 3 it is seen that, for the flat plate, the effect of the cutout is to increase the frequency and the increase is monotonic. The effect of curvature is to increase the frequency; for curved plates, there is a decrease in the value when the cutout size is increased. Also it is seen that the behaviour is the same for all plates when the cutout size is large. This behaviour should be expected, because when the cutout size is large, only the portion of the plate very near the support is present and its stiffness is large.

Almost the same observation can also be seen in Figure 4. Here, the non-dimensional value monotonically increases for a flat plate, and for curved panels the value initially decreases and then increases. The increase in frequency is much less than that of the flat plate when the plate has curvature and the ratio of the frequency of a plate with a cutout to that of a plate without a cut-out decreases with an increase in R/t.

The effect of curvature is to change the mode shape. A sample study was carried out on a square plate of $500\,\text{mm} \times 500\,\text{mm}$ with a cutout size of $250\,\text{mm} \times 250\,\text{mm}$ (symmetrical). Studies were also carried out on a curved panel of the same size with a radius of curvature of 500 mm (i.e., $R/t = 250$).

For plates without a cutout, the fundamental mode shape always has a single half wave in either direction; that is, there is no nodal point within the plate. This is seen for plates with any boundary condition. The cutout on the plate does not change this pattern, but the introduction of curvature does. With m denoting the number of half-waves in the longitudinal direction and n that in the circumferential direction, it is seen that, for all boundary conditions, the number of half-waves in the longitudinal direction is one and that in the circumferential direction is more than three. For the panels also, a cutout does

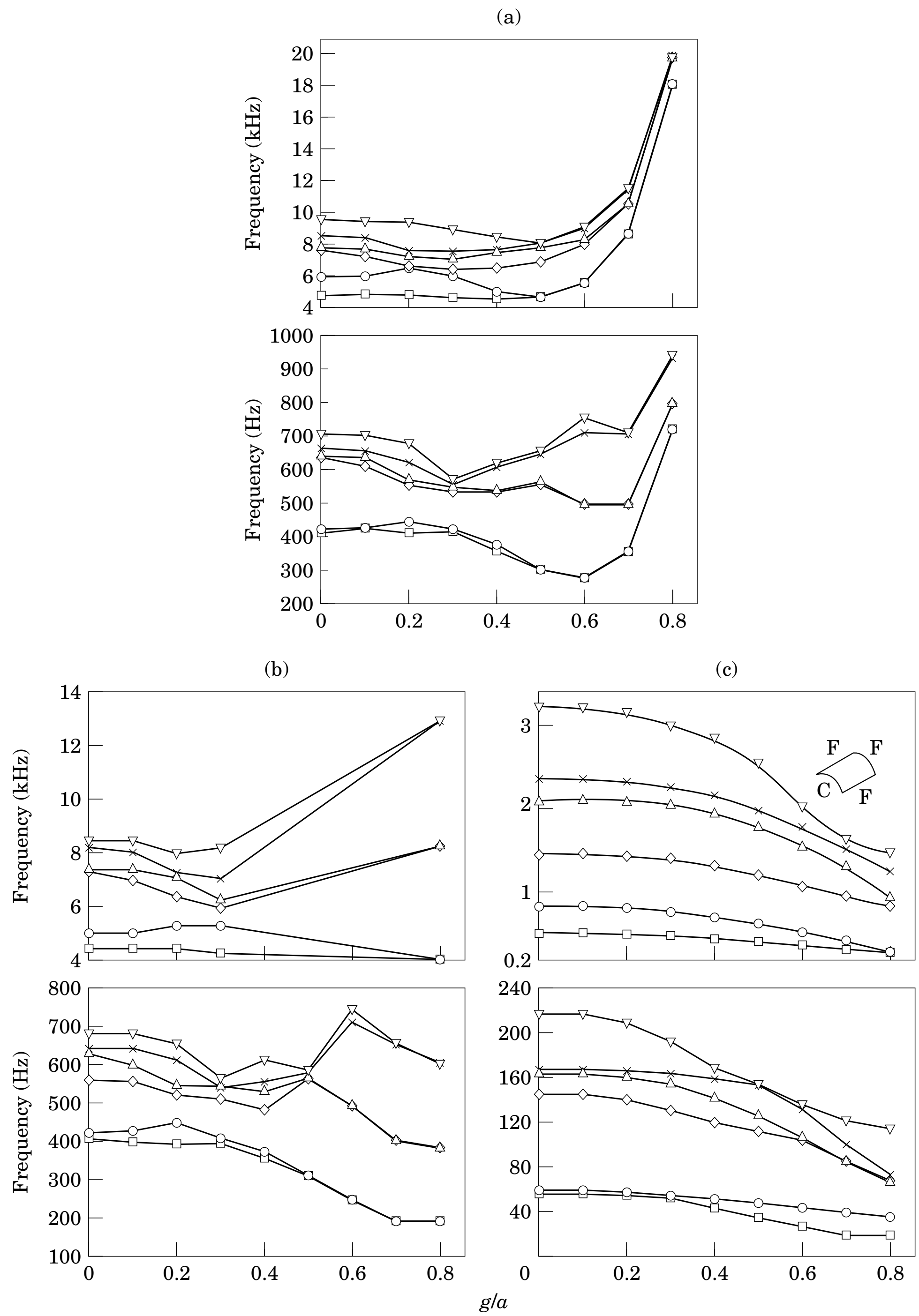

Figure 2(a), (b) and (c)—*continued overleaf*.

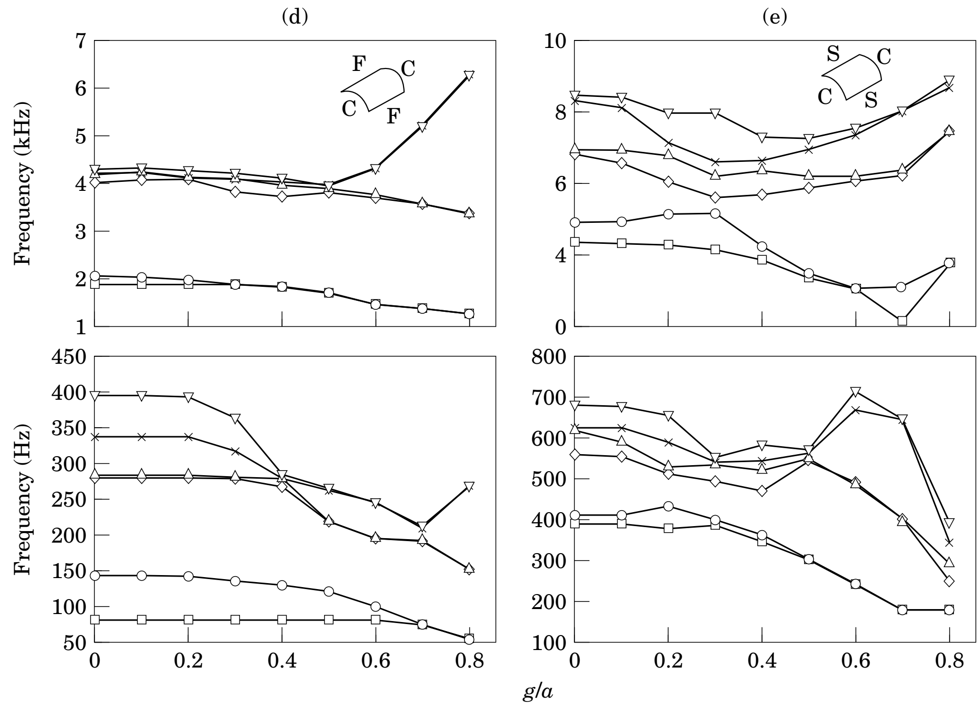

Figure 2. The variation of the first six natural frequencies with different cutout sizes for square panels. Boundary conditions: (a) CCCC; (b) SSSS; (c) CFFF; (d) CFCF; (e) CSCS. For top graphs, $R/t = 50$; for bottom graphs, $R/t = 250$. □, ○, ◇, △, ×, ▽, Modes 1–6 respectively.

not change the pattern much: i.e., the mode shapes corresponding to any frequency for curved panels with and without cutouts are either the same or interchanged between the two adjacent frequencies. There is a specific pattern for the mode shapes of plates with and without cutouts for higher frequencies: i.e., they will follow an ascending order such as (1, 1) (1, 2), (2, 1), (2, 2), etc. Such a pattern is not seen for panels with and without

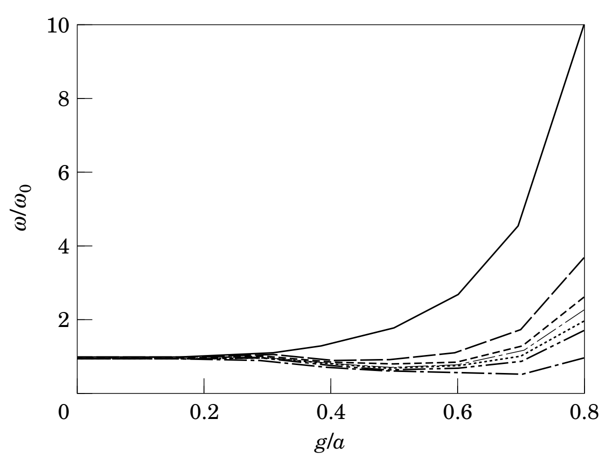

Figure 3. The ratio of the fundamental frequency of a CCCC panel with a cutout to that of the same panel without a cutout. ——, Plate; —·—, $R/t = 250$; – – – –, $R/t = 200$; —‑‑—, $R/t = 150$; ----, $R/t = 100$; —‑‑—‑‑—, $R/t = 75$; – · – · –, $R/t = 50$.

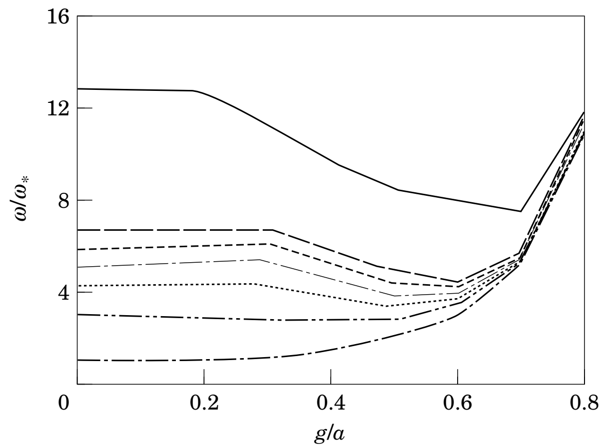

Figure 4. The effect of curvature on the fundamental frequency of a CCCC panel with cutouts. —·—·—, Plate; —··—··—, $R/t = 250$; -----, $R/t = 200$; —-–-—, $R/t = 150$; ——–, $R/t = 100$; —-—, $R/t = 75$; ——, $R/t = 50$.

TABLE 2

Mode shapes for the first five frequencies of plates and panels with and without cutout

Mode	Structure	Boundary condition CCCC	CFFF	CFCF	CSCS	SSSS
1	Pla I	1,1	1,1	1,1	1,1	1,1
	Pla II	1,1	1,1	1,1	1,1	1,1
	Pan I	1,3	1,3	1,4	1,4	1,4
	Pan II	1,4	1,2	1,2	1,4	1,4
2	Pla I	1,2	1,2	1,3	1,2	1,2
	Pla II	1,2	1,2	1,2	1,2	1,2
	Pan I	1,4	1,2	1,3	1,3	1,3
	Pan II	1,3	1,3	1,3	1,3	1,3
3	Pla I	2,1	2,1	1,3	2,1	2,1
	Pla II	2,1	2,1	2,1	2,1	2,1
	Pan I	1,5	1,4	2,4	1,5	1,5
	Pan II	2,4	1,4	1,5	1,4	1,4
4	Pla I	2,2	1,3	2,1	2,2	2,2
	Pla II	2,2	2,3	1,3	2,2	2,2
	Pan I	2,4	2,3	2,5	2,4	2,4
	Pan II	2,3	2,3	1,6	1,5	1,5
5	Pla I	3,3	2,2	2,2	1,3	1,3
	Pla II	2,3	2,2	2,1	3,1	3,1
	Pan I	2,5	2,4	1,5	2,5	2,5
	Pan II	1,4	2,1	2,4	2,4	2,4

Pla I, plate without cutout; Pla II, plate with cutout; Pan I, panel without cutout; Pan II, panel with cutout. (The first number corresponds to the number of half waves in the longitudinal direction and the second to that in the circumferential direction.)

cutouts. Mode shapes for the first five frequencies for plate and panel with and without cutouts for various boundary conditions are presented in Table 2.

REFERENCES

1. P. Paramasivam 1979 *Journal of Sound and Vibration* **30**, 173–178. Free vibration of square plate with square openings.
2. G. Aksu and R. Ali 1976 *Journal of Sound and Vibration* **44**, 147–158. Determination of dynamic characteristics of rectangular plates with cut-outs using a finite difference formulation.
3. G. Mundkur R. B. Bhat, and S. Neriya 1994 *Journal of Sound and Vibration* **176**, 136–144. Vibration of plates with cut-outs using boundary characteristic orthogonal polynomial functions in the Rayleigh–Ritz method.
4. M. Petyt and J. M. Debnath 1971 *Journal of Sound and Vibration* **13**, 485–497. Analysis of singly curved rectangular plates.
5. M. Petyt 1971 *Journal of Sound and Vibration* **15**, 381–395. Vibration of curved plates.
6. E. Szechenyi 1971 *Journal of Sound and Vibration* **14**, 401–408. Approximate methods for determination of the natural frequencies of stiffened and curved plates.
7. J. J. Webster and G. B. Warburton 1971 *Journal of Sound and Vibration* **18**, 139–145. Discussion of approximate methods for the determination of the natural frequencies of stiffened and curved plates.
8. M. D. Olson and G. M. Lindberg *MRC*, *NACE*, *CR*-497. A finite cylindrical shell element and vibrations of curved fan blades.
9. A. E. H. Love *A Treatise of the Mathematical Theory of Elasticity*. Cambridge: Cambridge University Press; fourth edition. See pp. 529–543.